CHAPTER 11

Fixed-Effect Model

INTRODUCTION

In this chapter we introduce the fixed-effect model. We discuss the assumptions of this model, and show how these are reflected in the formulas used to compute a summary effect, and in the meaning of the summary effect.

THE TRUE EFFECT SIZE

Under the fixed-effect model we assume that all studies in the meta-analysis share a common (true) effect size. Put another way, all factors that could influence the effect size are the same in all the studies, and therefore the true effect size is the same (hence the label *fixed*) in all the studies. We denote the true (unknown) effect size by theta (θ).

In Figure 11.1 the true overall effect size is 0.60 and this effect (represented by a triangle) is shown at the bottom. The true effect for each study is represented by a circle. Under the definition of a fixed-effect model the true effect size for each study must also be 0.60, and so these circles are aligned directly above the triangle.

IMPACT OF SAMPLING ERROR

Since all studies share the same true effect, it follows that the observed effect size varies from one study to the next only because of the random error inherent in each study. If each study had an infinite sample size the sampling error would be zero and the observed effect for each study would be the same as the true effect. If we were to plot the observed effects rather than the true effects, the observed effects would exactly coincide with the true effects.

Introduction to Meta-Analysis, Second Edition.
Michael Borenstein, Larry V. Hedges, Julian P.T. Higgins, and Hannah R. Rothstein.

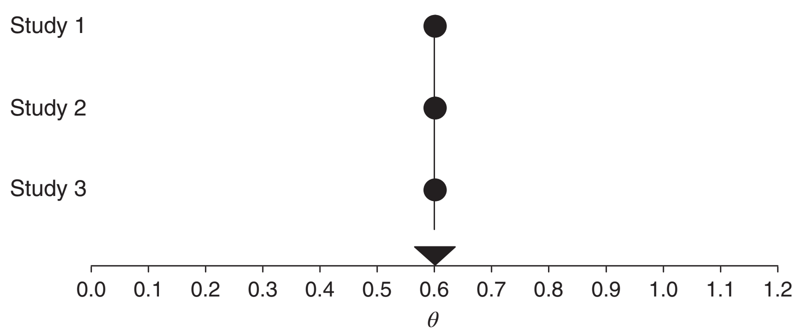

Figure 11.1 Fixed-effect model – true effects.

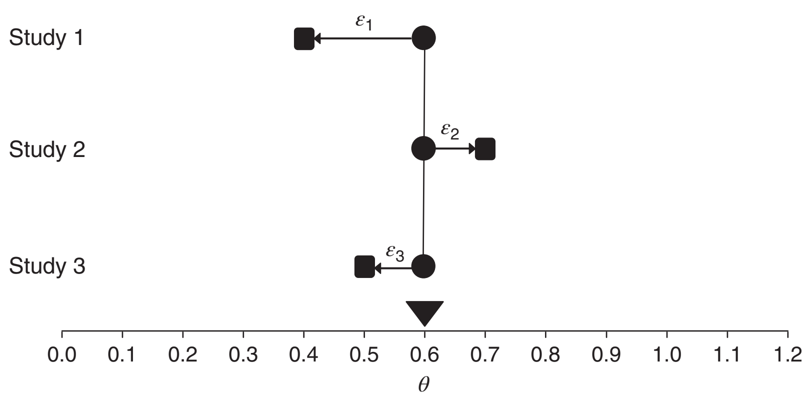

Figure 11.2 Fixed-effect model – true effects and sampling error.

In practice, of course, the sample size in each study is not infinite, and so there is sampling error and the effect observed in the study is not the same as the true effect. In Figure 11.2 the true effect for each study is still 0.60 (as depicted by the circles) but the observed effect (depicted by the squares) differs from one study to the next.

In Study 1 the sampling error (ε_1) is –0.20, which yields an observed effect (Y_1) of

$$Y_1 = 0.60 - 0.20 = 0.40.$$

In Study 2 the sampling error (ε_2) is 0.10, which yields an observed effect (Y_2) of

$$Y_2 = 0.60 + 0.10 = 0.70.$$

In Study 3 the sampling error (ε_3) is –0.10, which yields an observed effect (Y_3) of

$$Y_3 = 0.60 - 0.10 = 0.50.$$

More generally, the observed effect Y_i for any study is given by the population mean plus the sampling error in that study. That is,

$$Y_i = \theta + \varepsilon_i. \qquad (11.1)$$

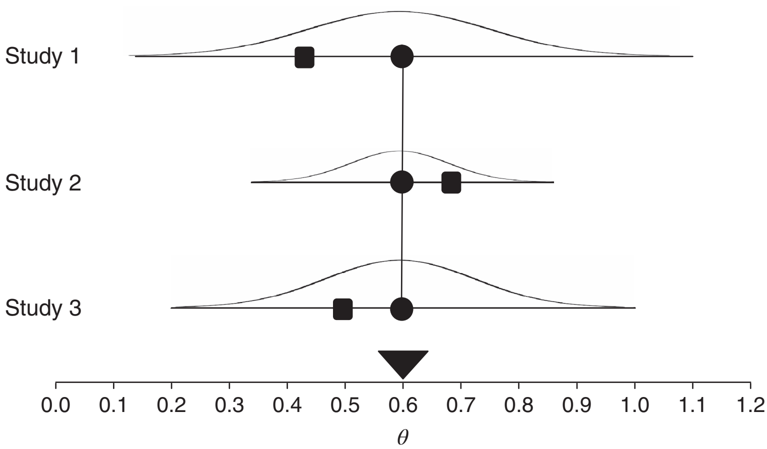

Figure 11.3 Fixed-effect model – distribution of sampling error.

While the error in any given study is random, we *can* estimate the sampling distribution of the errors. In Figure 11.3 we have placed a normal curve about the true effect size for each study, with the width of the curve being based on the variance in that study. In Study 1 the sample size was small, the variance large, and the observed effect is likely to fall anywhere in the relatively wide range of 0.20 to 1.00. By contrast, in Study 2 the sample size was relatively large, the variance is small, and the observed effect is likely to fall in the relatively narrow range of 0.40 to 0.80. (The width of the normal curve is based on the square root of the variance, or standard error).

PERFORMING A FIXED-EFFECT META-ANALYSIS

In an actual meta-analysis, of course, rather than starting with the population effect and making projections about the observed effects, we work backwards, starting with the observed effects and trying to estimate the population effect. In order to obtain the most precise estimate of the population effect (to minimize the variance) we compute a weighted mean, where the weight assigned to each study is the inverse of that study's variance. Concretely, the weight assigned to each study in a fixed-effect meta-analysis is

$$W_i = \frac{1}{V_{Y_i}}, \tag{11.2}$$

where V_{Y_i} is the within-study variance for study (i). The weighted mean (M) is then computed as

$$M = \frac{\sum_{i=1}^{k} W_i Y_i}{\sum_{i=1}^{k} W_i}, \tag{11.3}$$

that is, the sum of the products $W_i Y_i$ (effect size multiplied by weight) divided by the sum of the weights.

The variance of the summary effect is estimated as the reciprocal of the sum of the weights, or

$$V_M = \frac{1}{\sum_{i=1}^{k} W_i}, \tag{11.4}$$

and the estimated standard error of the summary effect is then the square root of the variance,

$$SE_M = \sqrt{V_M}. \tag{11.5}$$

Then, 95% lower and upper limits for the summary effect are estimated as

$$LL_M = M - 1.96 \times SE_M \tag{11.6}$$

and

$$UL_M = M + 1.96 \times SE_M. \tag{11.7}$$

Finally, a Z-value to test the null hypothesis that the common true effect θ is zero can be computed using

$$Z = \frac{M}{SE_M}. \tag{11.8}$$

For a one-tailed test the p-value is given by

$$p = 1 - \Phi(\pm|Z|), \tag{11.9}$$

where we choose '+' if the difference is in the expected direction and '–' otherwise, and for a two-tailed test by

$$p = 2[1 - (\Phi(|Z|))], \tag{11.10}$$

where $\Phi(Z)$ is the standard normal cumulative distribution. This function is tabled in many introductory statistics books, and is implemented in Excel as the function =NORMSDIST(Z).

Illustrative example

We suggest that you turn to a worked example for the fixed-effect model before proceeding to the random-effects model. A worked example for the standardized mean difference (Hedges' g) is on page 81, a worked example for the odds ratio is on page 85, and a worked example for correlations is on page 90.

SUMMARY POINTS

- Under the fixed-effect model all studies in the analysis share a common true effect.
- The summary effect is our estimate of this common effect size, and the null hypothesis is that this common effect is zero (for a difference) or one (for a ratio).
- All observed dispersion reflects sampling error, and study weights are assigned with the goal of minimizing this within-study error.