CHAPTER 14

Worked Examples (Part 1)

INTRODUCTION

In this chapter we present worked examples for continuous data (using the standardized mean difference), binary data (using the odds ratio) and correlational data (using the Fisher's z transformation).

All of the data sets and all computations are available as Excel spreadsheets on the book's website (www.Introduction-to-Meta-Analysis.com).

WORKED EXAMPLE FOR CONTINUOUS DATA (PART 1)

In this example we start with the mean, standard deviation, and sample size, and will use the bias-corrected standardized mean difference (Hedges' g) as the effect size measure.

Summary data

The summary data for six studies are presented in Table 14.1.

Compute the effect size and its variance for each study

The first step is to compute the effect size (g) and variance for each study using the formulas in Chapter 4 (see (4.18) to (4.24)). For the first study (Carroll) we compute the pooled within-groups standard deviation

$$S_{within} = \sqrt{\frac{(60-1)\times 22^2 + (60-1)\times 20^2}{60+60-2}} = 21.0238.$$

Introduction to Meta-Analysis, Second Edition.
Michael Borenstein, Larry V. Hedges, Julian P.T. Higgins, and Hannah R. Rothstein.

Table 14.1 Dataset 1 – Part A (basic data).

Study	Treated			Control		
	Mean	SD	*n*	Mean	SD	*n*
Carroll	94	22	60	92	20	60
Grant	98	21	65	92	22	65
Peck	98	28	40	88	26	40
Donat	94	19	200	82	17	200
Stewart	98	21	50	88	22	45
Young	96	21	85	92	22	85

Then we compute the standardized mean difference, *d*, and its variance as

$$d_1 = \frac{94 - 92}{21.0238} = 0.0951,$$

and

$$V_{d_1} = \frac{60 + 60}{60 \times 60} + \frac{0.0951^2}{2(60 + 60)} = 0.0334.$$

The correction factor (*J*) is estimated as

$$J = \left(1 - \frac{3}{4 \times 118 - 1}\right) = 0.9936.$$

Finally, the bias-corrected standardized mean difference, Hedges' *g*, and its variance are given by

$$g_1 = 0.9936 \times 0.0951 = 0.0945,$$

and

$$V_{g_1} = 0.9936^2 \times 0.0334 = 0.0329.$$

This procedure is repeated for all six studies.

Compute the summary effect using the fixed-effect model

The effect size and its variance are copied into Table 14.2 where they are assigned the generic labels Y and V_Y. We then compute the other values shown in the table. For Carroll,

$$W_1 = \frac{1}{0.0329} = 30.3515,$$

$$W_1 Y_1 = 30.3515 \times 0.0945 = 2.8690,$$

and so on for the other five studies. The sum of W is 244.215 and the sum of WY is 101.171. From these numbers we can compute the summary effect and related statistics, using formulas from Part 3 as follows (see (11.3) to (11.10)). In the computations that follow we use the generic M to represent Hedges' *g*.

$$M = \frac{101.171}{244.215} = 0.4143,$$

Table 14.2 Dataset 1 – Part B (fixed-effect computations).

Study	Effect size	Variance within	Weight	Calculated quantities		
	Y	V_Y	W	WY	WY^2	W^2
Carroll	0.095	0.033	30.352	2.869	0.271	921.214
Grant	0.277	0.031	32.568	9.033	2.505	1060.682
Peck	0.367	0.050	20.048	7.349	2.694	401.931
Donat	0.664	0.011	95.111	63.190	41.983	9046.013
Stewart	0.462	0.043	23.439	10.824	4.999	549.370
Young	0.185	0.023	42.698	7.906	1.464	1823.115
Sum			244.215	101.171	53.915	13802.325

$$V_M = \frac{1}{244.215} = 0.0041,$$

$$SE_M = \sqrt{0.0041} = 0.0640,$$

$$LL_M = 0.4143 - 1.96 \times 0.0640 = 0.2889,$$

$$UL_M = 0.4143 + 1.96 \times 0.0640 = 0.5397,$$

and

$$Z = \frac{0.4143}{0.0640} = 6.4739.$$

For a one-tailed test the p-value is given by

$$p = 1 - \Phi(6.4739) < 0.0001,$$

and for a two-tailed test, by

$$p = 2[1 - \Phi(|6.4739|)] < 0.0001.$$

In words, using fixed-effect weights, the standardized mean difference (Hedges' g) is 0.41 with a 95% confidence interval of 0.29 to 0.54. The Z-value is 6.47, and the p-value is <0.0001 (one-tailed) or <0.0001 (two tailed). These results are illustrated in Figure 14.1.

Compute an estimate of τ^2

To estimate τ^2, the variance of the true standardized mean differences, we use the DerSimonian and Laird method (see (12.2) to (12.5)). Using sums from Table 14.2,

$$Q = 53.915 - \left(\frac{101.171^2}{244.215}\right) = 12.0033,$$

$$df = (6 - 1) = 5,$$

$$C = 244.215 - \left(\frac{13802.325}{244.215}\right) = 187.698,$$

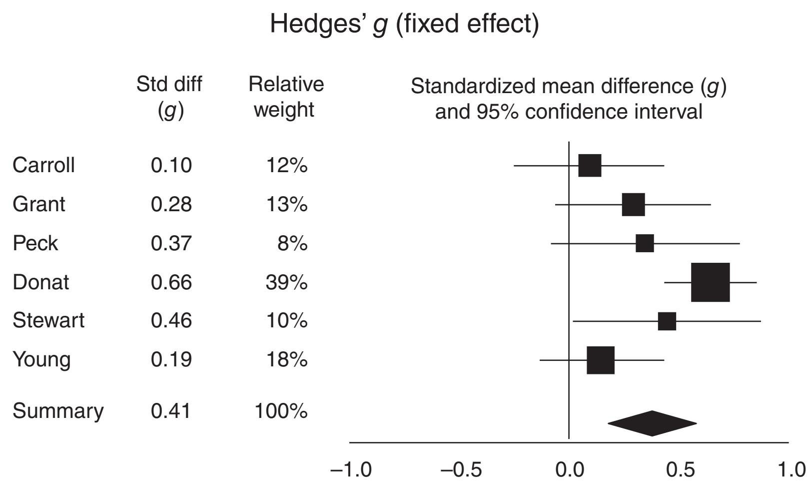

Figure 14.1 Forest plot of Dataset 1 – fixed-effect weights.

and

$$T^2 = \frac{12.0033 - 5}{187.698} = 0.0373.$$

Compute the summary effect using the random-effects model

To compute the summary effect using the random-effects model we use the same formulas as for the fixed effect, but the variance for each study is now the sum of the variance within studies plus the variance between studies (see (12.6) to (12.13)).

For Carroll,

$$W_1^* = \frac{1}{(0.0329 + 0.0373)} = \frac{1}{(0.070)} = 14.2331,$$

and so on for the other studies as shown in Table 14.3. Note that the within-study variance is unique for each study, but there is only one value of τ^2, so this value (estimated as 0.037) is applied to all studies.

Then,

$$M^* = \frac{32.342}{90.284} = 0.3582, \tag{14.1}$$

$$V_{M^*} = \frac{1}{90.284} = 0.0111, \tag{14.2}$$

$$SE_{M^*} = \sqrt{0.0111} = 0.1052,$$

$$LL_{M^*} = 0.3582 - 1.96 \times 0.1052 = 0.1520,$$

$$UL_{M^*} = 0.3582 + 1.96 \times 0.1052 = 0.5645,$$

$$Z^* = \frac{0.3582}{0.1052} = 3.4038,$$

Table 14.3 Dataset 1 – Part C (random-effects computations).

Study	Effect size Y	Variance within V_Y	Variance between T^2	Variance total $V_Y + T^2$	Weight W^*	Calculated quantities W^*Y
Carroll	0.095	0.033	0.037	0.070	14.233	1.345
Grant	0.277	0.031	0.037	0.068	14.702	4.078
Peck	0.367	0.050	0.037	0.087	11.469	4.204
Donat	0.664	0.011	0.037	0.048	20.909	13.892
Stewart	0.462	0.043	0.037	0.080	12.504	5.774
Young	0.185	0.023	0.037	0.061	16.466	3.049
Sum					90.284	32.342

and, for a one-tailed test

$$p^* = 1 - \Phi(3.4038) = 0.0003$$

or, for a two-tailed test

$$p^* = 2[1 - \Phi(|3.4038|)] = 0.0007.$$

In words, using random-effect weights, the standardized mean difference (Hedges' g) is 0.36 with a 95% confidence interval of 0.15 to 0.56. The Z-value is 3.40, and the p-value is 0.0003 (one-tailed) or 0.0007 (two-tailed). These results are illustrated in Figure 14.2.

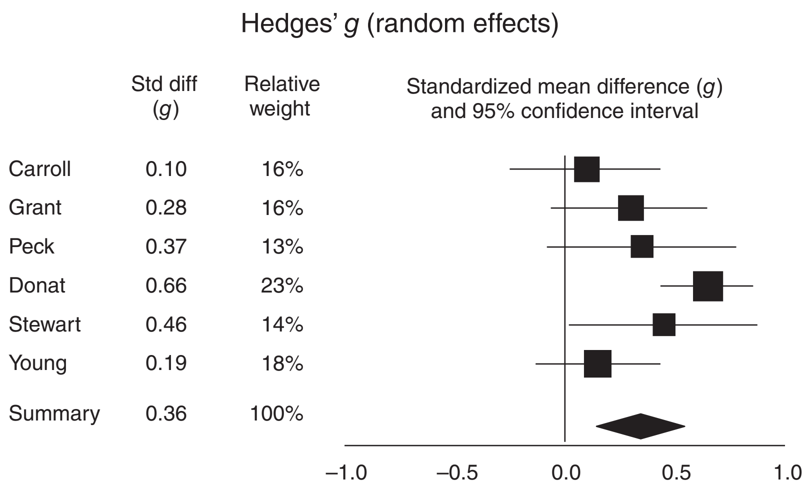

Figure 14.2 Forest plot of Dataset 1 – random-effects weights.

WORKED EXAMPLE FOR BINARY DATA (PART 1)

In this example we start with the events and non-events in two independent groups and will use the odds ratio as the effect size measure.

Table 14.4 Dataset 2 – Part A (basic data).

Study	Treated			Control		
	Events	Non-events	*n*	Events	Non-events	*n*
Saint	12	53	65	16	49	65
Kelly	8	32	40	10	30	40
Pilbeam	14	66	80	19	61	80
Lane	25	375	400	80	320	400
Wright	8	32	40	11	29	40
Day	16	49	65	18	47	65

Summary data

The summary data for six studies is presented in Table 14.4.

Compute the effect size and its variance for each study

For an odds ratio all computations are carried out using the log transformed values (see formulas (5.8) to (5.10)). For the first study (Saint) we compute the odds ratio, then the log odds ratio and its variance as

$$OddsRatio_1 = \frac{12 \times 49}{53 \times 16} = 0.6934,$$

$$LogOddsRatio_1 = \ln(0.6934) = -0.3662,$$

and

$$V_{LogOddsRatio_1} = \frac{1}{12} + \frac{1}{53} + \frac{1}{16} + \frac{1}{49} = 0.1851.$$

This procedure is repeated for all six studies.

Compute the summary effect using the fixed-effect model

The effect size and its variance (in log units) are copied into Table 14.5 where they are assigned the generic labels Y and V_Y.

For Saint

$$W_1 = \frac{1}{0.1851} = 5.4021,$$

$$W_1 Y_1 = 5.4021 \times (-0.3662) = -1.9780,$$

and so on for the other five studies.

The sum of W is 42.248 and the sum of WY is –30.594. From these numbers we can compute the summary effect and related statistics as follows (see (11.3) to (11.10)). In the computations that follow we use the generic M to represent the log odds ratio.

Table 14.5 Dataset 2 – Part B (fixed-effect computations).

Study	Effect size	Variance within	Weight	Calculated quantities		
	Y	V_Y	W	WY	WY^2	W^2
Saint	–0.366	0.185	5.402	–1.978	0.724	29.184
Kelly	–0.288	0.290	3.453	–0.993	0.286	11.925
Pilbeam	–0.384	0.156	6.427	–2.469	0.948	41.300
Lane	–1.322	0.058	17.155	–22.675	29.971	294.298
Wright	–0.417	0.282	3.551	–1.480	0.617	12.607
Day	–0.159	0.160	6.260	–0.998	0.159	39.190
Sum			42.248	–30.594	32.705	428.503

$$M = \frac{-30.594}{42.248} = -0.7241,$$

$$V_M = \frac{1}{42.248} = 0.0237,$$

$$SE_M = \sqrt{0.0237} = 0.1539,$$

$$LL_M = (-0.7241) - 1.96 \times 0.1539 = -1.0257,$$

$$UL_M = (-0.7241) + 1.96 \times 0.1539 = -0.4226,$$

and

$$Z = \frac{-0.7241}{0.1539} = -4.7068.$$

For a one-tailed test the p-value is given by

$$p = 1 - \Phi(-4.7068) < 0.0001,$$

and for a two-tailed test, by

$$p = 2[1 - \Phi(|-4.7068|)] < 0.0001.$$

We can convert the log odds ratio and confidence limits to the odds ratio scale using

$$OddsRatio = \exp(-0.7241) = 0.4847,$$

$$LL_{OddsRatio} = \exp(-1.0257) = 0.3586,$$

and

$$UL_{OddsRatio} = \exp(-0.4226) = 0.6553.$$

In words, using fixed-effect weights, the summary odd ratio is 0.48 with a 95% confidence interval of 0.36 to 0.66. The Z-value is –4.71, and the p-value is <0.0001 (one-tailed) or <0.0001 (two-tailed). These results are illustrated in Figure 14.3.

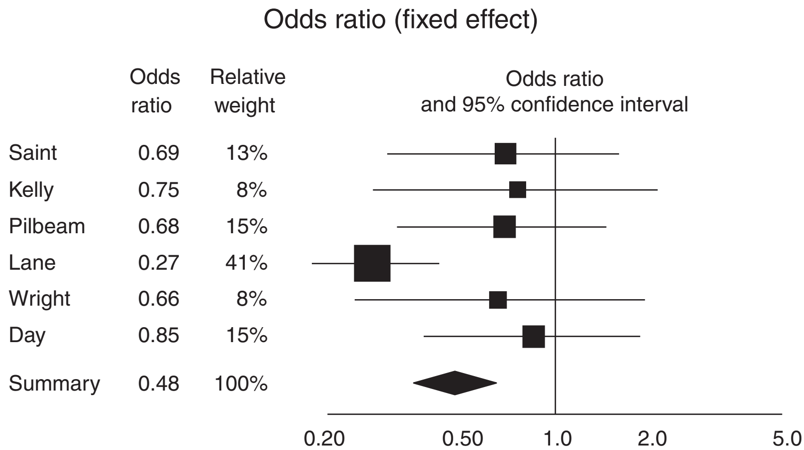

Figure 14.3 Forest plot of Dataset 2 – fixed-effect weights.

Compute an estimate of τ^2

To estimate τ^2, the variance of the true log odds ratios, we use the DerSimonian and Laird method (see (12.2) to (12.5)). Using sums from Table 14.5,

$$Q = 32.705 - \left(\frac{-30.594^2}{42.248}\right) = 10.5512,$$

$$df = (6 - 1) = 5,$$

$$C = 42.248 - \left(\frac{428.503}{42.248}\right) = 32.1052,$$

and,

$$T^2 = \frac{10.5512 - 5}{32.1052} = 0.1729.$$

These values are reported only on a log scale.

Compute the summary effect using the random-effects model

To compute the summary effect using the random-effects model, we use the same formulas as for the fixed effect, but the variance for each study is now the sum of the variance within studies plus the variance between studies (see (12.6) to (12.13)).

For Saint,

$$W_1^* = \frac{1}{(0.1851 + 0.1729)} = \frac{1}{(0.3580)} = 2.7932,$$

and so on for the other studies as shown in Table 14.6. Note that the within-study variance is unique for each study, but there is only one value of τ^2, so this value (estimated as 0.173) is applied to all studies.

Table 14.6 Dataset 2 – Part C (random-effects computations).

Study	Effect size Y	Variance within V_Y	Variance between T^2	Variance total $V_Y + T^2$	Weight W^*	Calculated quantities $W^* Y$
Saint	–0.366	0.185	0.173	0.358	2.793	–1.023
Kelly	–0.288	0.290	0.173	0.462	2.162	–0.622
Pilbeam	–0.384	0.156	0.173	0.329	3.044	–1.169
Lane	–1.322	0.058	0.173	0.231	4.325	–5.717
Wright	–0.417	0.282	0.173	0.455	2.200	–0.917
Day	–0.159	0.160	0.173	0.333	3.006	–0.479
Sum					17.531	–9.928

Then,

$$M^* = \frac{-9.928}{17.531} = -0.5663, \tag{14.3}$$

$$V_{M^*} = \frac{1}{17.531} = 0.0570, \tag{14.4}$$

$$SE_{M^*} = \sqrt{0.0570} = 0.2388,$$

$$LL_M = (-0.5663) - 1.96 \times 0.2388 = -1.0344,$$

$$UL_{M^*} = (-0.5663) + 1.96 \times 0.2388 = -0.0982,$$

$$Z^* = \frac{-0.5663}{0.2388} = -2.3711,$$

and, for a one-tailed test

$$p^* = 1 - \Phi(-2.3711) = 0.0089$$

or, for a two-tailed test

$$p^* = 2[1 - \Phi(|-2.3711|)] = 0.0177$$

We can convert the log odds ratio and confidence limits to the odds ratio scale using

$$OddsRatio^* = \exp(-0.5663) = 0.5676,$$

$$LL_{OddsRatio*} = \exp(-1.0344) = 0.3554,$$

and

$$UL_{OddsRatio*} = \exp(-0.0982) = 0.9065.$$

In words, using random-effects weights, the summary odds ratio is 0.57 with a 95% confidence interval of 0.36 to 0.91. The Z-value is –2.37, and the p-value is 0.0089 (one-tailed) or 0.0177 (two-tailed). These results are illustrated in Figure 14.4.

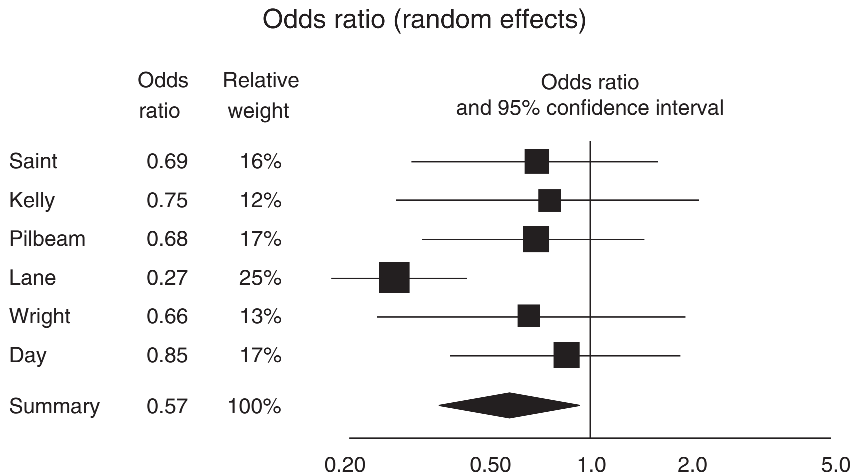

Figure 14.4 Forest plot of Dataset 2 – random-effects weights.

WORKED EXAMPLE FOR CORRELATIONAL DATA (PART 1)

Summary data

In this example we start with the correlation and sample size in six studies, as shown in Table 14.7.

Table 14.7 Dataset 3 – Part A (basic data).

Study	Correlation	*N*
Fonda	0.50	40
Newman	0.60	90
Grant	0.40	25
Granger	0.20	400
Milland	0.70	60
Finch	0.45	50

Compute the effect size and its variance for each study

For correlations, all computations are carried out using the Fisher's z transformed values (see formulas (6.2) to (6.3)). For the first study (Fonda) we compute the Fisher's z value and its variance as

$$z_1 = 0.5 \times \ln\left(\frac{1+0.50}{1-0.50}\right) = 0.5493,$$

and

$$V_1 = \frac{1}{40-3} = 0.0270.$$

This procedure is repeated for all six studies.

Compute the summary effect using the fixed-effect model

The effect size and its variance (in the Fisher's z metric) are copied into Table 14.8 where they are assigned the generic labels Y and V_Y.

For Fonda

$$W_1 = \frac{1}{0.0270} = 37.0000,$$

$$W_1 Y_1 = 37.000 \times (0.5493) = 20.3243,$$

and so on for the other five studies.

The sum of W is 647.000 and the sum of WY is 242.650. From these numbers we can compute the summary effect and related statistics as follows (see (11.3) to (11.10)). In the computations that follow we use the generic M to represent the Fisher's z score.

$$M = \frac{242.650}{647.000} = 0.3750,$$

$$V_M = \frac{1}{647.000} = 0.0015,$$

$$SE_M = \sqrt{0.0015} = 0.0393,$$

$$LL_M = 0.3750 - 1.96 \times 0.0393 = 0.2980,$$

$$UL_M = 0.3750 + 1.96 \times 0.0393 = 0.4521,$$

and

$$Z = \frac{0.3750}{0.0393} = 9.5396.$$

For a one-tailed test the p-value is given by

$$p = 1 - \Phi(9.5396) < 0.0001,$$

and for a two-tailed test, by

$$p = 2[1 - \Phi(|9.5396|)] < 0.0001.$$

Table 14.8 Dataset 3 – Part B (fixed-effect computations).

Study	Effect size	Variance within	Weight	Calculated quantities		
	Y	V_Y	W	WY	WY^2	W^2
Fonda	0.5493	0.0270	37.000	20.324	11.164	1369.000
Newman	0.6931	0.0115	87.000	60.304	41.799	7569.000
Grant	0.4236	0.0455	22.000	9.320	3.949	484.000
Granger	0.2027	0.0025	397.000	80.485	16.317	157609.000
Milland	0.8673	0.0175	57.000	49.436	42.876	3249.000
Finch	0.4847	0.0213	47.000	22.781	11.042	2209.000
Sum			647.000	242.650	127.147	172489.000

We can convert the effect size and confidence limits from the Fisher's z metric to correlations using

$$r = \frac{e^{(2\times0.3750)} - 1}{e^{(2\times0.3750)+1}} = 0.3584,$$

$$LL_r = \frac{e^{(2\times0.2980)} - 1}{e^{(2\times0.2980)} + 1} = 0.2895,$$

and

$$UL_r = \frac{e^{(2\times0.4521)} - 1}{e^{(2\times0.4521)} + 1} = 0.4236.$$

In words, using fixed-effect weights, the summary estimate of the correlation is 0.36 with a 95% confidence interval of 0.29 to 0.42. The Z-value is 9.54, and the p-value is <0.0001 (one-tailed) or <0.0001 (two tailed). These results are illustrated in Figure 14.5.

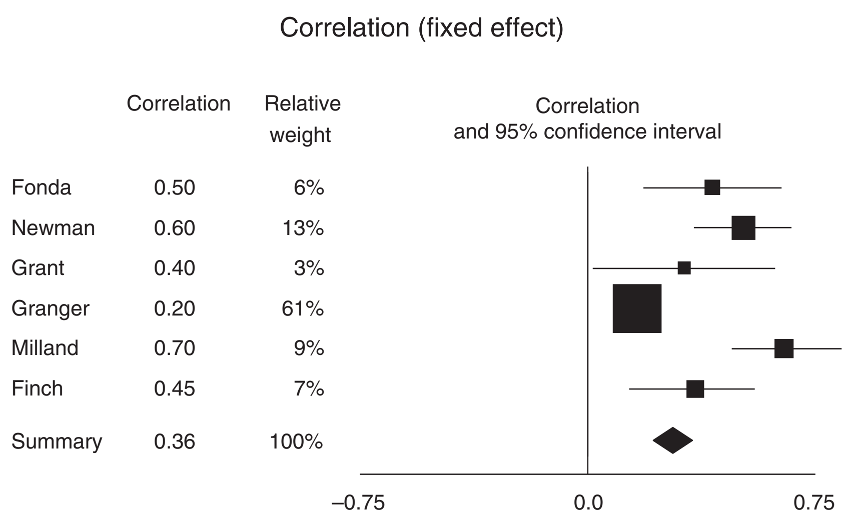

Figure 14.5 Forest plot of Dataset 3 – fixed-effect weights.

Compute an estimate of τ^2

To estimate τ^2, the variance of the true FIsher's z, we use the DerSimonian and Laird method (see (12.2) to (12.5)). Using sums from Table 14.8,

$$Q = 127.147 - \left(\frac{242.650^2}{647.000}\right) = 36.1437,$$

$$df = (6 - 1) = 5,$$

$$C = 647.000 - \left(\frac{172489.000}{647.000}\right) = 380.4019,$$

and

$$T^2 = \frac{36.1437 - 5}{380.4019} = 0.0819.$$

Compute the summary effect using the random-effects model

To compute the summary effect using the random-effects model, we use the same formulas as for the fixed effect, but the variance for each study is now the sum of the variance within studies plus the variance between studies (see (12.6) to (12.13)).

For Fonda,

$$W_1^* = \frac{1}{(0.0270 + 0.0819)} = \frac{1}{(0.1089)} = 9.1829,$$

and so on for the other studies as shown in Table 14.9. Note that the within-study variance is unique for each study, but there is only one value of τ^2, so this value (estimated as 0.0819) is applied to all studies.

Then,

$$M^* = \frac{31.621}{59.351} = 0.5328, \quad (14.5)$$

$$V_{M^*} = \frac{1}{59.351} = 0.0168, \quad (14.6)$$

$$SE_{M^*} = \sqrt{0.0168} = 0.1298,$$

$$LL_{M^*} = (0.5328) - 1.96 \times 0.1298 = 0.2784,$$

$$UL_{M^*} = (0.5328) + 1.96 \times 0.1298 = 0.7872,$$

and

$$Z^* = \frac{0.5328}{0.1298} = 4.1045.$$

Then, for a one-tailed test

$$p^* = 1 - \Phi(4.1045) < 0.0001,$$

or, for a two-tailed test

$$p^* = 2[1 - \Phi(|4.1045|)] < 0.0001.$$

Table 14.9 Dataset 3 – Part C (random-effects computations).

Study	Effect size Y	Variance within V_Y	Variance between T^2	Variance total $V_Y + T^2$	Weight W^*	Calculated quantities $W^* Y$
Fonda	0.549	0.027	0.082	0.109	9.183	5.044
Newman	0.693	0.012	0.082	0.093	10.711	7.424
Grant	0.424	0.046	0.082	0.127	7.854	3.327
Granger	0.203	0.003	0.082	0.084	11.850	2.402
Milland	0.867	0.018	0.082	0.099	10.059	8.724
Finch	0.485	0.021	0.082	0.103	9.695	4.699
Sum					59.351	31.621

We can convert the effect size and confidence limits from the Fisher's z metric to correlations using

$$r^* = \frac{e^{(2\times0.5328)} - 1}{e^{(2\times0.5328)} + 1} = 0.4875,$$

$$LL_{r^*} = \frac{e^{(2\times0.2784)} - 1}{e^{(2\times0.2784)} + 1} = 0.2714,$$

and

$$UL_{r^*} = \frac{e^{(2\times0.7872)} - 1}{e^{(2\times0.7872)} + 1} = 0.6568.$$

In words, using random-effects weights, the summary estimate of the correlation is 0.49 with a 95% confidence interval of 0.27 to 0.66. The Z-value is 4.10, and the p-value is <0.0001 (one-tailed) or <0.0001 (two tailed). These results are illustrated in Figure 14.6.

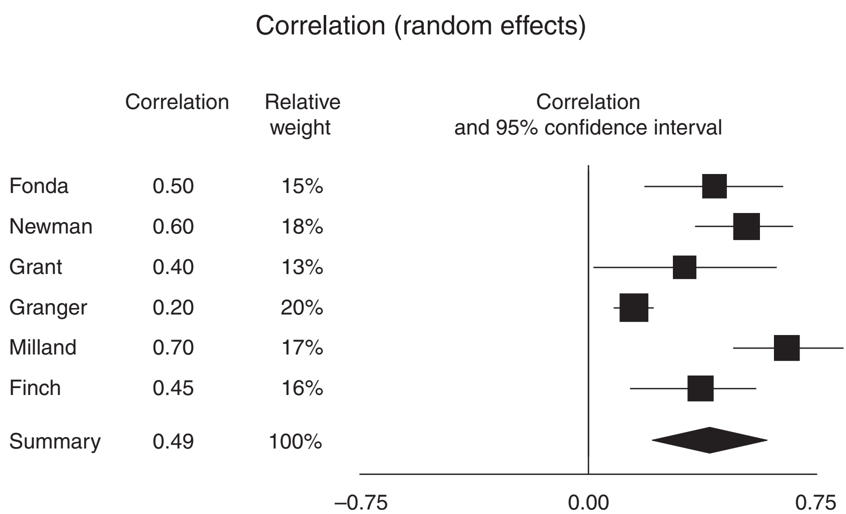

Figure 14.6 Forest plot of Dataset 3 – random-effects weights.

SUMMARY POINTS

- This chapter includes worked examples showing how to compute the summary effect using fixed-effect and random-effects models.
- For the standardized mean difference we work with the effect sizes directly.
- For ratios we work with the log transformed data.
- For correlations we work with the Fisher's z transformed data.
- These worked examples are available as Excel files on the book's website (www .Introduction-to-Meta-Analysis.com).