CHAPTER 18

Worked Examples (Part 2)

INTRODUCTION

In Chapter 14 we presented worked examples for computing a summary effect using continuous, binary, and correlational data. Here, we continue with the same three data sets and show how to compute the measures of heterogeneity discussed in Chapters 16 and 17.

These computations are also included in Excel spreadsheets that can be downloaded from the book's website

WORKED EXAMPLE FOR CONTINUOUS DATA (PART 2)

On page 81 we showed how to compute the effect size and variance for each study. Here, we proceed from that point.

Using results in Table 18.1, the summary effect is given by

$$M = \frac{101.171}{244.215} = 0.4143,$$

which value is used in the column labeled *Mean* in Table 18.2.

Then, using (16.1) we sum the values in the final column of Table 18.2,

$$Q = \sum_{i=1}^{k} W_i (Y_i - M)^2 = 12.0033.$$

Or, using (12.3) and results in Table18.1,

$$Q = 53.915 - \frac{(101.171)^2}{244.215} = 12.0033.$$

Introduction to Meta-Analysis, Second Edition.
Michael Borenstein, Larry V. Hedges, Julian P.T. Higgins, and Hannah R. Rothstein.

Table 18.1 Dataset 1 – Part D (intermediate computations).

Study	Effect	Variance	Weight	Calculated quantities			
	Y	V_Y	W	WY	WY^2	W^2	W^3
Carroll	0.095	0.033	30.352	2.869	0.271	921.21	27960.25
Grant	0.277	0.031	32.568	9.033	2.505	1060.68	34544.41
Peck	0.367	0.050	20.048	7.349	2.694	401.93	8058.00
Donat	0.664	0.011	95.111	63.190	41.983	9046.01	860371.10
Stewart	0.462	0.043	23.439	10.824	4.999	549.37	12876.47
Young	0.185	0.023	42.698	7.906	1.464	1823.12	77843.29
Sum			244.215	101.171	53.915	13802.33	1021653.52

Table 18.2 Dataset 1 – Part E (variance computations).

Study	Effect	Variance	Weight	Mean	Calculated quantities	
	Y	V_Y	W	M	$(Y-M)^2$	$W(Y-M)^2$
Carroll	0.095	0.033	30.352	0.414	0.102	3.103
Grant	0.277	0.031	32.568	0.414	0.019	0.610
Peck	0.367	0.050	20.048	0.414	0.002	0.046
Donat	0.664	0.011	95.111	0.414	0.063	5.950
Stewart	0.462	0.043	23.439	0.414	0.002	0.053
Young	0.185	0.023	42.698	0.414	0.052	2.241
Sum						12.003

Under the assumption that all studies share a common effect, the expected value of Q is given by

$$df = 6 - 1 = 5$$

where k is the number of studies. The difference,

$$12.003 - 5 = 7.0033,$$

is the excess value which we attribute to differences in the true effect sizes.

The p-value for $Q = 12.003$ with $df = 5$, is 0.035. In Excel, the function =CHIDIST(12.003,5) returns 0.035. If we are using 0.10 or 0.05 as the criterion for statistical significance, we would reject the null hypothesis that all the studies share a common effect size, and accept the alternative, that the true effect is not the same in all studies.

Then, using formulas (16.6), (16.5), (16.8), and (16.9),

$$C = 244.215 - \left(\frac{13802.33}{244.215}\right) = 187.6978,$$

$$T^2 = \frac{12.003 - 5}{187.698} = 0.0373,$$

$$T = \sqrt{0.0373} = 0.1932,$$

and

$$I^2 = \left(\frac{12.003 - 5}{12.003}\right) \times 100\% = 58.34\%.$$

To compute the standard error of T^2 (from (16.11) to (16.13)), we have $sw1 = 244.215$, $sw2 = 13{,}802.33$, and $sw3 = 1{,}021{,}653.52$, so that

$$A = \left[df + 2\left(244 - \frac{13802}{244}\right)0.0373 + \left(13802 - 2\left(\frac{1021653}{244}\right) + \frac{(13802)^2}{(244)^2}\right)0.0373^2\right] = 31.0202.$$

Then, the variance of T^2 is

$$V_{T^2} = 2 \times \left(\frac{31.020}{187.698^2}\right) = 0.0018,$$

and its standard error is given by

$$SE_{T^2} = \sqrt{0.0018} = 0.0420.$$

Since Q = 12.003 > 6 = (df + 1), we compute, from (16.14) to (16.19),

$$B = 0.5 \times \frac{\ln(12.0033) - \ln(5)}{\sqrt{2 \times 12.0033} - \sqrt{2 \times 5 - 1}} = 0.2305.$$

Then compute intermediate values

$$L = \mathrm{Exp}\left(0.5 \times \ln\left(\frac{12.003}{5}\right) - 1.96 \times 0.2305\right) = 0.9862$$

and

$$U = \mathrm{Exp}\left(0.5 \times \ln\left(\frac{12.003}{5}\right) + 1.96 \times 0.2305\right) = 2.4343.$$

Finally, the 95% confidence intervals for τ^2 may be obtained as

$$LL_{T^2} = \frac{5 \times (0.9862^2 - 1)}{187.698} = -0.0007,$$

which is set to zero, and

$$UL_{T^2} = \frac{5 \times (2.4343^2 - 1)}{187.698} = 0.1312.$$

The 95% confidence interval for τ may be obtained by taking the square roots of the confidence limits for τ^2, namely

$$LL_T = \sqrt{0.0} = 0.0,$$

and

$$UL_T = \sqrt{0.1312} = 0.3622.$$

Confidence intervals for I^2

Since 12.003 > (5 + 1) we compute, using formulas (16.20) through (16.25),

$$B = 0.5 \times \frac{\ln\,(12.003) - \ln\,(5)}{\sqrt{2 \times 12.003} - \sqrt{2 \times 5 - 1}} = 0.2305.$$

Compute intermediate values

$$L = \exp\left(0.5 \times \ln\left(\frac{12.003}{5}\right) - 1.96 \times 0.2305\right) = 0.9862$$

and

$$U = \exp\left(0.5 \times \ln\left(\frac{12.003}{5}\right) + 1.96 \times 0.2305\right) = 2.4343.$$

The 95% confidence intervals may then be obtained as

$$LL_{I^2} = \left(\frac{0.9862^2 - 1}{0.9862^2}\right) \times 100\% = -2.82\%,$$

which is set to zero, and

$$UL_{I^2} = \left(\frac{2.4343^2 - 1}{2.4343^2}\right) \times 100\% = 83.12.\%.$$

To obtain a 95% prediction interval for the true standardized mean difference in a future study, we use the random-effects weighted mean and its variance computed in (14.1) and (14.2), $M^* = 0.3582$ and $V_{M^*} = 0.0111$ and compute, from (17.7) and (17.8),

$$t_4^{0.05} = 2.7764,$$

$$LL_{pred} = 0.3582 - 2.7764 \times \sqrt{0.0373 + 0.0111} = -0.2525,$$

and

$$UL_{pred} = 0.3582 + 2.7764 \times \sqrt{0.0373 + 0.0111} = 0.9690.$$

This prediction interval is plotted is in Figure 18.1.

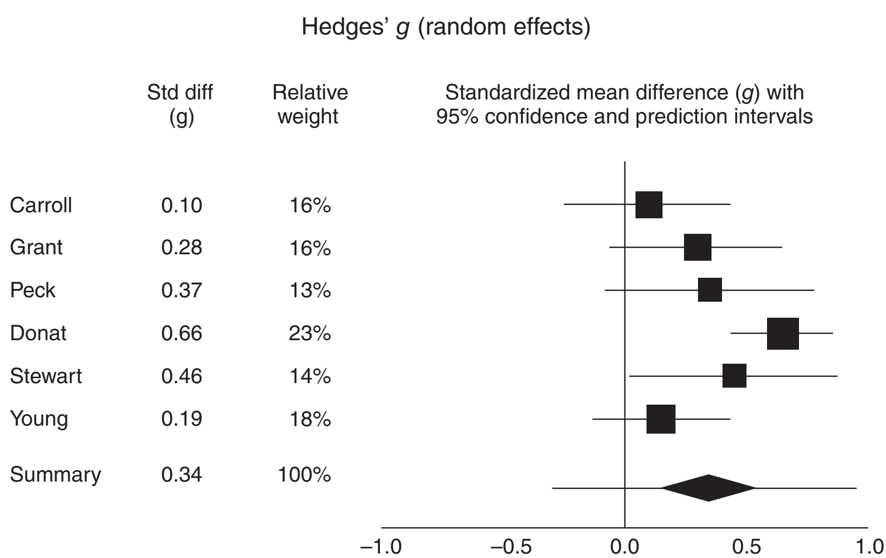

Figure 18.1 Forest plot of Dataset 1 – random-effects weights with prediction interval.

WORKED EXAMPLE FOR BINARY DATA (PART 2)

On page 85 we showed how to compute the effect size (here, the log odds ratio) and variance for each study. Here, we proceed from that point.

Using results in Table 18.3, the summary effect is given by

$$M = \frac{-30.594}{42.248} = -0.7241,$$

which value is used in the column labeled *Mean* in Table 18.4.

Then, using (16.1) we sum the values in the final column of Table 18.4,

$$Q = \sum_{i=1}^{k} W_i (Y_i - M)^2 = 10.5512.$$

Or, using (12.3) and results in Table 18.3,

$$Q = 32.705 - \frac{(-30.594)^2}{42.248} = 10.5512.$$

Under the assumption that all studies share a common effect, the expected value of Q is given by

$$df = 6 - 1 = 5,$$

Table 18.3 Dataset 2 – Part D (intermediate computations).

Study	Effect	Variance	Weight	Calculated quantities			
	Y	V_Y	W	WY	WY^2	W^2	W^3
Saint	–0.366	0.185	5.402	–1.978	0.724	29.18	157.66
Kelly	–0.288	0.290	3.453	–0.993	0.286	11.92	41.18
Pilbeam	–0.384	0.156	6.427	–2.469	0.948	41.30	265.42
Lane	–1.322	0.058	17.155	–22.675	29.971	294.30	5048.71
Wright	–0.417	0.282	3.551	–1.480	0.617	12.61	44.76
Day	–0.159	0.160	6.260	–0.998	0.159	39.19	245.33
Sum			42.248	–30.594	32.705	428.50	5803.06

Table 18.4 Dataset 2 – Part E (variance computations).

Study	Effect	Variance	Weight	Mean	Calculated quantities	
	Y	V_Y	W	M	$(Y-M)^2$	$W(Y-M)^2$
Saint	–0.366	0.185	5.402	–0.724	0.128	0.692
Kelly	–0.288	0.290	3.453	–0.724	0.191	0.658
Pilbeam	–0.384	0.156	6.427	–0.724	0.116	0.743
Lane	–1.322	0.058	17.155	–0.724	0.357	6.127
Wright	–0.417	0.282	3.551	–0.724	0.094	0.335
Day	–0.159	0.160	6.260	–0.724	0.319	1.996
Sum						10.551

where k is the number of studies. The difference,

$$10.5512 - 5 = 5.5512$$

is the excess value which we attribute to differences in the true effect sizes.

The p-value for $Q = 10.551$ with $df = 5$, is 0.0610. In Excel, the function =CHIDIST (10.551,5) returns 0.0610. If we are using 0.10 as the criterion for statistical significance, we would reject the null hypothesis that all the studies share a common effect size, and accept the alternative, that the true effect is not the same in all studies. If we are using 0.05 as the criterion, we would not have sufficient evidence to reject the null hypothesis (but would not conclude that the effects are homogeneous, since the nonsignificant p-value could be due to inadequate statistical power).

Then, using formulas (16.6), (16.5), (16.8), and (16.9),

$$C = 42.248 - \left(\frac{428.50}{42.248}\right) = 32.1052,$$

$$T^2 = \frac{10.5512 - 5}{32.1052} = 0.1729,$$

$$T = \sqrt{0.1729} = 0.4158,$$

and

$$I^2 = \left(\frac{10.5512 - 5}{10.5512}\right) \times 100 = 52.61\%.$$

To compute the standard error of T^2 (from (16.11) to (16.13)), we have $sw1 = 42.25$, $sw2 = 428.5$, and $sw3 = 5{,}803.1$, so that

$$A = \left[df + 2\left(42.25 - \frac{428.5}{42.25}\right)0.1729 + \left(428.5 - 2\left(\frac{5803.1}{42.25}\right) + \frac{(428.5)^2}{(42.25)^2}\right)0.1729^2\right] = 23.7754.$$

Then, the variance of T^2 is

$$V_{T^2} = 2 \times \left(\frac{23.7754}{32.1052^2}\right) = 0.0461$$

and its standard error is given by

$$SE_{T^2} = \sqrt{0.0461} = 0.2148.$$

Since $Q = 10.5512 > 6\ 5\ (df + 1)$, we compute, from (16.14) to (16.19),

$$B = 0.5 \times \frac{\ln(10.5512) - \ln(5)}{\sqrt{2 \times 10.5512} - \sqrt{2 \times 5 - 1}} = 0.2343.$$

Then compute intermediate values

$$L = \mathrm{Exp}\left(0.5 \times \ln\left(\frac{10.5512}{5}\right) - 1.96 \times 0.2343\right) = 0.9178$$

and

$$U = \mathrm{Exp}\left(0.5 \times \ln\left(\frac{10.5512}{5}\right) + 1.96 \times 0.2343\right) = 2.2993.$$

Finally, the 95% confidence intervals for τ^2 may then be obtained as

$$LL_{T^2} = \frac{5 \times (0.9178^2 - 1)}{32.1052} = -0.0246,$$

which is set to zero, and

$$UL_{T^2} = \frac{5 \times (2.2993^2 - 1)}{32.1052} = 0.6676.$$

The 95% confidence interval for τ may be obtained by taking the square roots of the confidence limits for τ^2, namely

$$LL_T = \sqrt{0.0} = 0.0,$$

and

$$UL_T = \sqrt{0.6676} = 0.8171.$$

Confidence intervals for I^2

Since 10.5512 > (5 + 1) we compute, using formulas (16.20) through (16.25),

$$B = 0.5 \times \frac{\ln(10.5512) - \ln(5)}{\sqrt{2 \times 10.5512} - \sqrt{2 \times 5 - 1}} = 0.2343,$$

then compute intermediate values

$$L = \text{Exp}\left(0.5 \times \ln\left(\frac{10.5512}{5}\right) - 1.96 \times 0.2343\right) = 0.9178$$

and

$$U = \text{Exp}\left(0.5 \times \ln\left(\frac{10.5512}{5}\right) + 1.96 \times 0.2343\right) = 2.2993.$$

The 95% confidence intervals may then be obtained as

$$LL_{I^2} = \left(\frac{0.9178^2 - 1}{0.9178^2}\right) \times 100\% = -18.72\%,$$

which is set to zero, and

$$UL_{I^2} = \left(\frac{2.2993^2 - 1}{2.2993^2}\right) \times 100\% = 81.09\%.$$

To obtain a 95% prediction interval for the true log odds ratio in a future study, we use the random-effects weighted mean and its variance computed in (14.3) and (14.4), $M^* = -0.5663$ and $V_{M^*} = 0.0570$, and compute, from (17.7) and (17.8),

$$t_4^{0.05} = 2.7764,$$

$$LL_{pred} = -0.5663 - 2.7764 \times \sqrt{0.1729 + 0.0570} = -1.8977,$$

and

$$UL_{pred} = -0.5663 + 2.7764 \times \sqrt{0.1729 + 0.0570} = 0.7651.$$

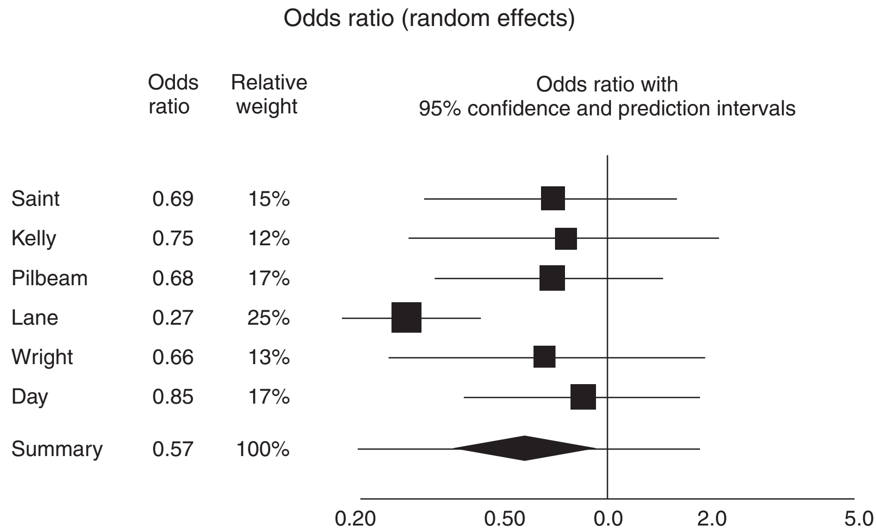

Figure 18.2 Forest plot of Dataset 2 – random-effects weights with prediction interval.

These limits are computed on a log scale. We can convert the limits to the odds ratio scale using

$$LL_{pred} = \exp(-1.8977) = 0.1499$$

and

$$UL_{pred} = \exp(0.7651) = 2.1492.$$

This prediction interval is plotted in Figure 18.2.

WORKED EXAMPLE FOR CORRELATIONAL DATA (PART 2)

On page 90 we showed how to compute the effect size (here, the Fisher's z transformation of the correlation coefficient) and variance for each study. Here, we proceed from that point.

Using results in Table 18.5, the summary effect is given by

$$M = \frac{242.650}{647.000} = 0.3750,$$

which value is used in the column labeled *Mean* in Table 18.6.

Then, using (16.1) we sum the values in the final column of Table 18.6,

$$Q = \sum_{i=1}^{k} W_i (Y_i - M)^2 = 36.1437.$$

Or, using (12.3) and results in Table 18.5,

$$Q = 127.147 - \frac{(242.650)^2}{647.000} = 36.1437.$$

Table 18.5 Dataset 3 – Part D (intermediate computations).

Study	Effect	Variance	Weight	Calculated quantities			
	Y	V_Y	W	WY	WY^2	W^2	W^3
Fonda	0.549	0.027	37.000	20.324	11.164	1369.00	50653.00
Newman	0.693	0.011	87.000	60.304	41.799	7569.00	658503.00
Grant	0.424	0.045	22.000	9.320	3.949	484.00	10648.00
Granger	0.203	0.003	397.000	80.485	16.317	157609.00	62570773.00
Milland	0.867	0.018	57.000	49.436	42.876	3249.00	185193.00
Finch	0.485	0.021	47.000	22.781	11.042	2209.00	103823.00
Sum			647.000	242.650	127.147	172489.00	63579593.00

Table 18.6 Dataset 3 – Part E (variance computations).

Study	Effect	Variance	Weight	Mean	Calculated quantities	
	Y	V_Y	W	M	$(Y-M)^2$	$W(Y-M)^2$
Fonda	0.549	0.027	37.000	0.375	0.030	1.124
Newman	0.693	0.011	87.000	0.375	0.101	8.804
Grant	0.424	0.045	22.000	0.375	0.002	0.052
Granger	0.203	0.003	397.000	0.375	0.030	11.787
Milland	0.867	0.018	57.000	0.375	0.242	13.812
Finch	0.485	0.021	47.000	0.375	0.012	0.565
Sum						36.144

Under the assumption that all studies share a common effect, the expected value of Q is given by

$$df = 6 - 1 = 5,$$

where k is the number of studies. The difference,

$$36.1437 - 5 = 31.1437,$$

is the excess value which we attribute to differences in the true effect sizes.

The p-value for $Q = 36.1437$ with $df = 5$, is less than 0.0001. In Excel, the function =CHIDIST(36.1437,5) returns < 0.0001. If we are using 0.10 or 0.05 as the criterion for statistical significance, we would reject the null hypothesis that all the studies share a common effect size, and accept the alternative, that the true effect is not the same in all studies.

Then, using formulas (16.6), (16.5), (16.8), and (16.9),

$$C = 647.000 - \left(\frac{172489.00}{647.000}\right) = 380.4019,$$

$$T^2 = \frac{36.1437 - 5}{380.4019} = 0.0819,$$

$$T = \sqrt{0.0819} = 0.28613,$$

and

$$I^2 = \left(\frac{36.1437 - 5}{36.1437}\right) \times 100 = 86.17\%.$$

To compute the standard error of T^2 (from (16.11) to (16.13)), we have $sw1 = 647.00$, $sw2 = 172489.00$, and $sw3 = 63{,}579{,}593.00$, so that

$$A = \left[df + 2\left(647.00 - \frac{172489}{647.00}\right)0.0819 + \left(172489 - 2\left(\frac{63579593}{647.00}\right) + \frac{(172489)^2}{(647.00)^2}\right)0.0819^2\right] = 382.4983.$$

Then, the variance of T^2 is

$$V_{T^2} = 2 \times \left(\frac{382.4983}{380.4019^2}\right) = 0.0053,$$

and its standard error is given by

$$SE_{T^2} = \sqrt{0.0053} = 0.0727.$$

Since $Q = 36.1437 > 6 = (df + 1)$, we compute, from (16.14) to (16.19),

$$B = 0.5 \times \frac{\ln(36.1437) - \ln(5)}{\sqrt{2 \times 36.1437} - \sqrt{2 \times 5 - 1}} = 0.1798.$$

Then compute intermediate values

$$L = \text{Exp}\left(0.5 \times \ln\left(\frac{36.1437}{5}\right) - 1.96 \times 0.1798\right) = 1.8903$$

and

$$U = \text{Exp}\left(0.5 \times \ln\left(\frac{36.1437}{5}\right) + 1.96 \times 0.1798\right) = 3.8242.$$

Finally, the 95% confidence intervals for τ^2 may then be obtained as

$$LL_{T^2} = \frac{5 \times (1.890^2 - 1)}{380.4019} = 0.0338$$

and

$$UL_{T^2} = \frac{5 \times (3.8242^2 - 1)}{380.4019} = 0.1791.$$

The 95% confidence interval for τ may be obtained by taking the square roots of the confidence limits for τ^2, namely

$$LL_T = \sqrt{0.0338} = 0.1839,$$

and

$$UL_T = \sqrt{0.1791} = 0.4232.$$

Confidence intervals for I^2

Since $Q = 36.1437 > 6 = (df + 1)$, we compute, from (16.20),

$$B = 0.5 \times \frac{\ln(36.1437) - \ln(5)}{\sqrt{2 \times 36.1437} - \sqrt{2 \times 5 - 1}} = 0.1798,$$

then compute intermediate values

$$L = \text{Exp}\left(0.5 \times \ln\left(\frac{36.1437}{5}\right) - 1.96 \times 0.1798\right) = 1.8903$$

and

$$U = \text{Exp}\left(0.5 \times \ln\left(\frac{36.1437}{5}\right) + 1.96 \times 0.1798\right) = 3.8242.$$

The 95% confidence intervals may then be obtained as

$$LL_{I^2} = \left(\frac{1.8903^2 - 1}{1.8903^2}\right) \times 100\% = 72.01\%,$$

and

$$UL_{I^2} = \left(\frac{3.8241^2 - 1}{3.8241^2}\right) \times 100\% = 93.16\%.$$

To obtain a 95% prediction interval for the true Fisher's z in a future study, we use the random-effects weighted mean and its variance computed in (14.5) and (14.6), $M^* = 0.5328$ and $V_{M*} = 0.0168$ and compute, from (17.7) and (17.8),

$$t_4^{0.05} = 2.7764,$$

$$LL_{pred} = 0.5328 - 2.7764 \times \sqrt{0.0819 + 0.0168} = -0.3396,$$

and

$$UL_{pred} = 0.5328 + 2.7764 \times \sqrt{0.0819 + 0.0168} = 1.4051.$$

These limits are in the Fisher's z metric. We can convert the limits to the correlation scale using

$$LL_{pred} = \frac{e^{(2\times -0.3396)} - 1}{e^{(2\times -0.3396)} + 1} = -0.3271$$

and

$$UL_{pred} = \frac{e^{(2\times 1.4051)} - 1}{e^{(2\times 1.4051)+1}} = 0.8865.$$

This prediction interval is plotted in Figure 18.3.

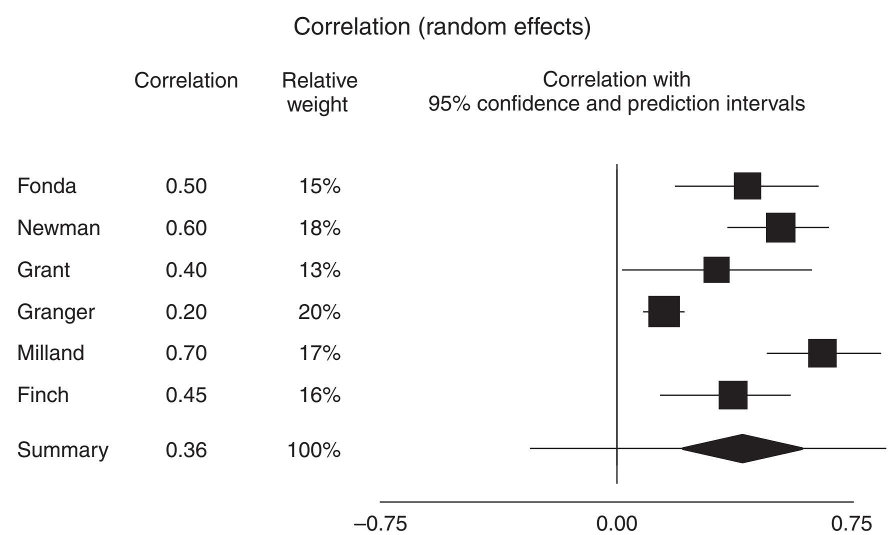

Figure 18.3 Forest plot of Dataset 3 – random-effects weights with prediction interval.

SUMMARY POINTS

- This chapter includes worked examples showing how to compute the summary effect using fixed-effect and random-effects models.
- For the standardized mean difference we work with the effect sizes directly.
- For ratios we work with the log transformed data.
- For correlations we work with the Fisher's z transformed data.
- These worked examples are available as Excel files on the book's website (www .Introduction-to-Meta-Analysis.com).